Discussion [0b10] [2_{10}] [0x2]: Number Representation
SOLUTIONS

Conversion

(a) Convert the following binary numbers into decimal.

11001 → **25**

1001001 → **73**

(b) Convert the following decimal numbers into binary.

12 → **0b1100**

64 → **0b1000000**

127 → **0b1111111**

(c) Convert the following binary numbers into hex.

10011001 → **0x99**

11110111 → **0xF7**

110000001111111111101110 → **0xC0FFEE**

Decimal	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Limits

(a) What is the biggest number that can be represented with two decimal digits?

99

(b) What is the biggest number that can be represented with three binary digits?

The greatest number that can be represented with three binary digits is 0b111. In decimal, this is $7 = 1 \times 2^0 + 1 \times 2^1 + 1 \times 2^2$
Note: Another way to do this is to notice that 0b111 is just 1 less than

0b1000, which is equal to 2^3. That means the greatest number that can be represented with three binary digits is $2^3 - 1 = 7$.

(c) What is the biggest number that can be represented with four hexadecimal digits?

The greatest number that can be represented with four hex digits is 0xFFFF. Following the note from part (b), we get $16^4 - 1 = 65535$.

(d) How many different numbers can you represent using three binary digits?

8 distinct numbers. The maximum number we can represent is 7, and the minimum number is 0.

More Conversion Practice

Fill in the blanks.

Decimal	Binary	Hexadecimal
12	**1100**	C
5	**101**	5
11	1011	**B**
25	11001	**19**
17	10001	11
27	11011	1B
8	**1000**	**8**
14	1110	**E**
30	**11110**	1E
73	**1001001**	49

Challenge Problems

(a) The original Pokemon are numbered 1-150. We want to store a binary encoding for all original Pokemon where each Pokemon has a binary code equivalent to their decimal number. How many bits do we need to use?

8 bits at minimum. The largest number that can be represented with 7 bits is 127 ($2^7 - 1$), and the largest number that can be represented with 8 bits is 255 ($2^8 - 1$). Since 150 is between these two numbers, we need 8 bits.

(b) What is the encoding for Pikachu (#25)?

0b11001

(c) Ternary utilizes base 3 instead of base 2. For example, 10 in ternary is equivalent to 3 in decimal. Imagine that we wanted to store a ternary encoding for all 150 Pokemon where each Pokemon has a ternary code equivalent to their decimal number. What is the ternary encoding for Pikachu (#25)?

221.
Using the method shown in lecture for converting decimal to other bases, we can create the following table:

Power	3^3	3^2	3^1	3^0
Value	27	9	3	1
Ternary digit				

Now, we notice that the greatest value listed in our table that is still less than 25 is 9. When we divide 25 by 9, we get 2 (with some remainder). Then, we will put a 2 in the ternary digit under 9, and our new value we are working with is 25 - (9 x 2) = 7. Our table then looks like this:

Power	3^3	3^2	3^1	3^0
Value	27	9	3	1
Ternary digit		2		

Now, we repeat the process with the value of 7. 3 is the greatest value that is less than 7, 7 divided by 3 gives us 2. So we put a 2 in the ternary digit under 3, and we continue with a value of 7 - (3 x 2) = 1. Our table looks like this:

Power	3^3	3^2	3^1	3^0
Value	27	9	3	1
Ternary digit		2	2	

Now, our new value is 1. The greatest value that's less than 1 is 1, and 1 divided by 1 is 1. Our new value is now 1-1 = 0, which means we are done. Our table looks as follows:

Power	3^3	3^2	3^1	3^0
Value	27	9	3	1
Ternary digit		2	2	1

And our answer is 221.

In some discussion sections, we introduced an alternative method for converting between bases. Below we demonstrate how we could use it to get the same answer as above.
(Note: if your TA didn't cover this method, don't worry about it. You only need to know about the lecture method (above) to solve exam problems. But if you're interested in learning the alternate method, come to OH or post on Piazza.)

New Base	Value	Remainder
3	25	

Every step of the way, we divide our value by the new base, and then record the remainder. So, at the next step, our table would look like this:

New Base	Value	Remainder
3	25	
3	8	1

We continue this process until our value is equal to 0.

New Base	Value	Remainder
3	25	
3	8	1
3	2	2
3	0	2

To get our answer, we read the remainder column upwards, so our answer is 221.